Dynamical Clusterization in the Presence of Instabilities

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Using a recently developed Boltzmann-Langevin treatment, we study cluster formation in unstable nuclear matter. The self-consistent propagation of the spontaneous fluctuations restores the predictive power of the one-body approach, even when such catastrophic phenomena are addressed.

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Many-body systems far from equilibrium are encountered in many areas of physics and the development of quantitatively useful approximate methods for describing their dynamical evolution forms an important part of modern physics. The complete dynamical treatment of many-body systems requires the consideration of the correlated evolution of a large number of degrees of freedom and is usually impractical. One avenue towards reducing the problem to a tractable form is to retain only a small portion of the dynamical information [1]. In general, the resulting effective equation of evolution for the retained variables will be nonlinear and so the associated dynamics may exhibit chaotic features, such as bifurcations and instabilities. Moreover, the specification of the retained variables characterizes an entire ensemble of microscopically different states of the many-body system. Therefore, the reduced description naturally admits concepts from nonequilibrium statistical mechanics, for example, entropy and irreversibility. Moreover, the fact that the microstate is incompletely specified introduces a stochastic element in the evolution of the projected state, so that the development of the retained variables resembles Brownian motion, with the missing information in effect acting as a heat bath.

In nuclear dynamics, as in many other areas of physics, it often suffices to retain only the reduced one-body density matrix. In the mean-field approximation, the fluctuations arising from the stochastic part of the evolution are ignored and a single average effective one-body density is considered. The approximation is most useful when the particular ensemble involved displays only little diversity, as is usually the case at relatively low degrees of agitation. But when widely different manifestations of the system are dynamically accessible the relevant ensemble is multimodal and, accordingly, it is meaningless to employ a single "average" representative one-particle density. This problem is especially evident when the dynamical evolution contains branch points from which very different further developments can occur, such as when the system enters an unstable region. Under such circumstances, one might be tempted to abandon the one-body approach altogether, in favor of a true many-body approach. However, a much simpler approach consists in retaining the stochastic element in the one-body evolution. Even though the system is then still represented in terms of a mean-field configuration, at any particular time, the stochastic nature of the reduced equation of motion enables the various members of the ensemble to experience different histories. It should also be noted that while the occurrence of instabilities renders the mean-trajectory method incapable of making reliable predictions, the inclusion of an entire ensemble of different dynamical evolutions restores the predictive power of the one-body approach.

In this paper we present a first application of a recently developed nuclear Boltzmann-Langevin (BL) treatment to a catastrophic process, namely, the clusterization of matter at subsaturation density. Although the approach was developed within the context of nuclear dynamics, the method is quite general and may therefore be of much broader utility.

Our starting point is the so-called Boltzmann-Uehling-Uhlenbeck (BUU) equation, which provides a semiclassical evolution of the one-nucleon phase-space density $f(\mathbf{r},\mathbf{p},t)$ taking into account the propagation in the mean field together with the average effect of Pauli-blocked two-body collisions between individual nucleons [2]. This approach was first developed by Nordheim for the description of electrons in a solid [3]. Recently, the BUU theory has been extensively applied to nuclear collisions and has proven fairly successful in describing one-body observables [4]. However, the model cannot provide a description for large fluctuation phenomena, such as multifragmentation, because it determines only an "average" dynamical history.

Therefore, it has been proposed to extend the model by including the fluctuating part of the collision integral [5–9]. In particular, in Ref. [8] we have proposed and tested a specific implementation of the BL equation,

$$\frac{\partial f}{\partial t}+\{H[f],f\}=I[f]\,, \tag{1}$$

where $H[f]$ is the self-consistent mean-field Hamiltonian and $I[f]$ is the stochastic collision term. The collision term produces sudden branchings of the dynamical histories, resulting in a bundle of trajectories. By contrast, the effective Hamiltonian leads to a smooth evolution. It is therefore convenient to consider the two evolutions sep-

arately.

In order to solve the above equation of motion, we represent $f(\mathbf{r},\mathbf{p})$ on a lattice of grid points in phase space. The size of each lattice cell is given by $\Delta s = \Delta \mathbf{r} \Delta \mathbf{p}/h^D$, D being the dimension of the space, and the value f_K at the lattice point $s_K = (\mathbf{r}_K, \mathbf{p}_K)$ represents the average value of f over the cell K. Since each physical particle represents a phase-space volume of h^D, at least, we first divide the phase space into cells K of the order of unity.

During a given small time interval, Δt, the expected number of elementary collisions $\bar{N}$ from the phase-space cells $K=1$ and 2 into the cells $1'$ and $2'$ is given by

$$\bar{N}_{1,2;1'2'} = f_1 f_2 \bar{f}_{1'} \bar{f}_{2'} \omega_{1,2;1'2'} \Delta s^4 \Delta t \,, \tag{2}$$

where ω is the elementary transition rate containing the energy and momentum conservation and the Pauli blocking factors $\bar{f} \equiv 1 - f$ express the availability of the final one-particle states.

The actual number of elementary transitions N is a stochastic variable, having a Poisson distribution characterized by the above mean value $\bar{N}$ and a variance $\sigma_N^2 = \bar{N}$. Therefore, the statistical properties of $I[f]$ are thus fully determined by the mean transition rates $\bar{N}$. In our numerical implementation, we simulate the distribution of N by a Gaussian with the variance $\sigma_N^2 = \bar{N}$. That this indeed provides a numerically accurate treatment of the Langevin term was recently illustrated [10].

The collisionless (Vlasov) part of the evolution is made by means of a standard matrix technique. However, in order to achieve sufficient accuracy a relatively fine lattice is required and we therefore further divide each physical cell K into sufficiently small subcells k of volume δs. The actual number of collisions N_{IJKL} is then shared between the subcells in proportion to the mean changes $\bar{n}_{ijkl}$, which are given in analogy with Eq. (2), so that $n_{ijkl} = N_{IJKL} \bar{n}_{ijkl} / \bar{N}_{IJKL}$. This method introduces the appropriate correlation of the noise over volumes Δs, thus ensuring that the relation between mean and variance is preserved on the physical scale, and it has the numerical advantage that both the collision term and the mean-field propagation are computed with the same small cells, thus avoiding degradation in accuracy.

As is generally the case with lattice calculations, the result is useful only for extracting observables that are smooth over the domain of a lattice cell. Fortunately, Planck's constant provides a lower limit on the physically relevant resolution in phase space, and the employed lattice spacings (see below) appear to be fully adequate in the context of our present studies.

We have studied a gas of fermions situated on a two-dimensional torus. For the effective one-body field we employ a simplified Skyrme interaction, $U(x) = A\bar{\rho}(x)/\rho_0 + B\bar{\rho}(x)^2/\rho_0^2$, with $A = -100.3$ MeV and $B = 48$ MeV. Moreover, $\rho_0 = 0.55$ fm^{-2} is the saturation density and $\bar{\rho}(x)$ is the average of the density $\rho(x,y)$ with respect to the transverse direction y. Since the effective field U then depends on x only, the same holds for the accessible modes, and this simplifies the analysis considerably. In order to mimic standard three-dimensional matter, we have required a Fermi momentum of $P_F = 260$ MeV/c, and a binding energy of 16 MeV/nucleon, and that a density doubling lead to approximately zero binding (corresponding to a compressibility modulus of $K \approx 300$ MeV for a calculation in three dimensions). It should be noted that the finite size of the spatial lattice emulates the effect of a finite interaction range, as has been explicitly verified.

We then solved the stochastic BUU equation on a lattice of 21 cells in the x direction and 25×25 cells in momentum space. The physical cells had a spatial resolution of $\Delta x = 1$ fm, corresponding approximately to the range of the nuclear force. Since there is no dependence on y, the length in the y direction was chosen to be large ($L_y = 5000$ fm). The momentum resolution was 65 MeV/c, so that the momentum space was extending up to $p_x = p_y = \pm 682.5$ MeV/c. The subcells were identical to the macrocells, except for the x direction for which the accuracy of the gradients requires a minimum of $\delta x = \frac{1}{3}$ fm. An accurate calculation of the Vlasov propagation was ensured by employing the relative short time increment $\delta t = 0.5$ fm/c. The range of the two-body interaction entering in collision integral was taken as 1.2 fm, and the longer time steps $\Delta t = 5$ fm/c were sufficient for obtaining a reliable calculation of this contribution to the evolution.

We have initialized the system as a uniform gas at half the saturation density, with Fermi-Dirac occupancies corresponding to the small temperature 3 MeV; this is in the mechanically unstable regime of the phase diagram. We analyze an ensemble of 100 trajectories; for each trajectory, the calculation was stopped at 90 fm/c. We have verified that our results and conclusions are robust against variations of the numerical parameters.

In Fig. 1 we show the time evolution of the density

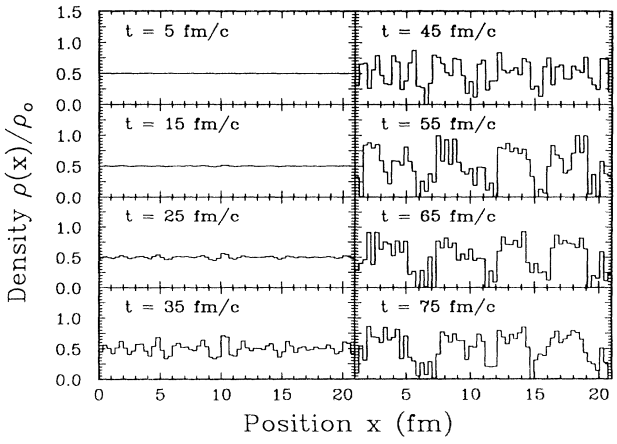

FIG. 1. The density profile $\bar{\rho}(x,t)$ associated with one particular trajectory vs the position x shown at eight different times t.

versus the position for one trajectory of the ensemble. Initially the system has a uniform density, but soon the fluctuations are breaking this initial symmetry. Subsequently, the fluctuations are rapidly amplified by the action of the effective one-body field, thus leading towards fragment formation. We note that the density distribution of each particular system keeps changing with time, with the clusters exhibiting fusion and fission. In this manner, each particular trajectory explores the various accessible configurations, while the distribution of trajectories quickly approaches the appropriate statistical limit.

To get a deeper insight into the mechanism of fragmentation and the role of the instabilities, we can study the onset of the phenomenon within the linear response theory. If the system is unstable, some eigenmodes will be exponentially amplified with a characteristic time τ [11]. We have computed the actual dispersion relation by analyzing the BUU evolution (without fluctuations) of a uniform density at 3 MeV temperature perturbed with a small (1%) harmonic variation in the x direction. In Fig. 2 we show the evolution of $1/\tau$ versus the wave number k of the eigenmode. As mentioned above, the lattice discretization simulates a finite range of the effective interaction and we have chosen a value corresponding to a physically reasonable range, which suffices for our present illustrative purposes. In a more refined approach, one should employ a finite-range interaction functional and perform the corresponding convolution at each time step, thus rendering results that remain unchanged as the lattice spacing is reduced.

This normal mode analysis will allow us to study the interplay between instabilities and fluctuations. Indeed, considering the Fourier transform of the fluctuations of the density, $F(k,t)=\int dx\, e^{i2\pi kx}\delta\rho(x,t)$, we find

$$\sigma_k(t)=\langle|F(k,t)|^2\rangle = \int\int dx\,dx'\, e^{i2\pi k(x-x')}\langle\delta\rho(x,t)\delta\rho(x',t)\rangle . \quad (3)$$

Within the linear response regime and for a given instability mode k, the random two-body collisions act as a continuous source of fluctuation C_k. Therefore the variance σ_k evolves according to the equation

$$\dot{\sigma}_k = C_k + (2/\tau_k)\sigma_k , \quad (4)$$

where the second term is due to the unstable character of the normal mode k, which has a growth time τ_k. The solutions of this equation are given by

$$\sigma_k(t) = \tfrac{1}{2} C_k \tau_k (e^{2t/\tau_k} - 1) + \sigma_k(0) e^{2t/\tau_k} , \quad (5)$$

where $\sigma_k(0)$ is the initial fluctuation, which is zero in the present case. The characteristic amplitude of the fluctuations is determined by C_k, and their time scale by τ_k. This solution, Eq. (5), provides a good understanding of the growth of the fluctuations. Figure 3 displays the fluctuation σ_k versus the reduced wave number k. At the early stage of the evolution ($t < \tau_k/2$), one mainly observes the Fourier component of the noise, $\sigma_k(t) \simeq C_k t$, and indeed we find that the spectrum at $t = 5$ fm/c is characteristic of a system where the fluctuations must be correlated over a domain of at least 1 fm in size. In particular, the spectrum does not extend beyond $k = 1$ fm^{-1}, and is symmetric around $k = 0.5$ fm^{-1}, corresponding to a spectrum on a lattice with a spacing of 1 fm. As time goes on, we observe the interplay of the stochastic collisions and the exponentially increasing propagation due to the unstable effective field.

The continual action of both agencies shifts the peak to slightly higher frequencies (from 0.5 to 0.6 fm^{-1}). However, it should be noted that the response above 1 fm^{-1} remains nearly zero throughout this initial stage and that the system is never fully dominated by the instabilities. This demonstrates that the system is keeping some memory of the physical processes that have induced the fluctuations.

After 30 fm/c the system enters into a nonlinear re-

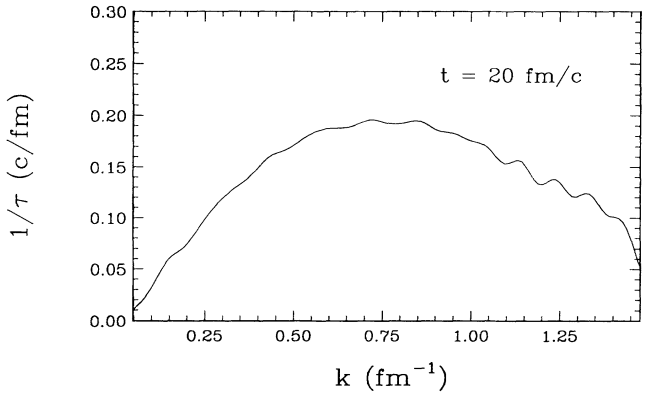

FIG. 2. The growth rate $1/\tau_k$ of harmonic modes with reduced wave number k in two-dimensional nuclear matter at half the saturation density, as resulting from our numerical implementation of the BUU model.

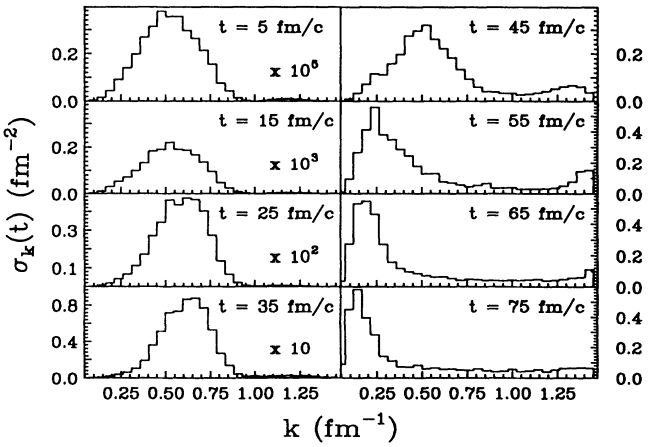

FIG. 3. The variance σ_k vs the reduced wave number k, shown at a number of times t.

gime, with frequency doubling leading soon to an irregular transition stage, before reaching an equilbrium characterized by a statistical population of "fragment" configurations. This equilibrium behavior is independent of the specific early evolution, as we have explicitly verified by making an alternate calculation starting with an uncorrelated random noise on the one-body density. In an actual nuclear collision, the expansion of the system will effectively truncate the evolution after a finite time, and the resulting fragment mass distribution will in general depend on the specific character of the fluctuations employed. Consequently, the proper treatment of the fluctuations is important and one may hope that multifragment observables may provide an informative basis for direct confrontation between theory and experiment.

In conclusion, using a recently developed nuclear Boltzmann-Langevin model, we have made a first dynamical simulation of a catastrophic evolution leading towards the multifragmentation of an initially uniform system. This calculation is based on a stochastic one-body description. The theory we have used is an extension of the Nordheim-type transport theories that have been widely used in recent years for the study of heavy-ion reactions. The essential new feature of the BL model is that it permits the spontaneous breaking of symmetries, which is essential for producing catastrophic phenomena, such as clusterization. While the ordinary theory is deterministic and is unable to make reliable predictions in the presence of instabilities, the inclusion of the stochastic term restores the predictive power of the theory. Such a description in terms of an ensemble of trajectories is closely related to statistical mechanics and is therefore able to accomodate a high degree of dynamical branching.

In the presented results, we have been able to discern two dynamical regimes: an early linear regime characterized by a competition between the stochastic creation of fluctuations and their exponential evolution due to the instability of the effective field, and a later complex one where the system behaves in an irregular manner as it seeks to condense into fragments. Since the detailed evolution towards equilibrium has been found to be sensitive to the specific treatment of the fluctuations, it appears that the inclusion of fluctuations is important for the quantitative description of fragment production in nuclear collisions, and consequently it may be expected that actual collision experiments can provide a means for testing the theory.

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